



Binary and Hexadecimal numbers

Computer and binary numbers

- Inside the computer, the binary string (bit string) is used for information processing
 - Based on the use of high and low voltages to process information, using the binary numbers is easier than using the decimal numbers to make a calculation
 - To calculate decimal numbers, first, they are converted to binary numbers. The binary numbers are then calculated. Finally, the resulting binary numbers are reconverted to decimal numbers for displaying.

Binary number

- Using 0 and 1

| | | | | | | | | |
|-----------------------|---|---|----|----|-----|-----|-----|-----|
| Decimal number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Binary number | 0 | 1 | 10 | 11 | 100 | 101 | 110 | 111 |

Radix (or Base)

- Radix of Decimal number is 10
 - $207_{(10)} = 2 \times 10^2 + 0 \times 10^1 + 7 \times 10^0$
 - The first digit is $10^0 (= 1)$
 - The second digit is 10^1
 - The third digit is 10^2
- Radix of binary number is 2
 - $101_{(2)} = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
 - The first digit is $2^0 (= 1)$
 - The second digit is 2^1

Using binary number to represent the weather

- One card has front as 0 and back as 1 value.
- To represent 4 types of weather, 2-digit binary number is needed.

| | |
|--------|----|
| sunny | 00 |
| rainy | 01 |
| snowy | 10 |
| cloudy | 11 |

The relation between the binary number and the amount of information

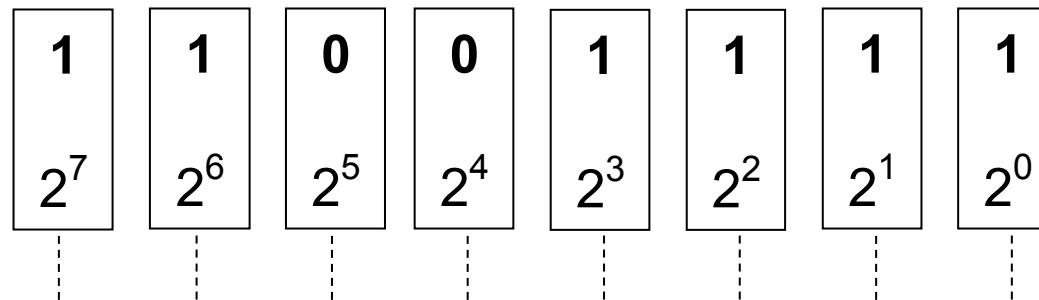
- One-digit binary number
 - distinguishes 2 kinds of information
- Two-digit binary number
 - distinguishes 4 kinds of information
- Three-digit binary number
 - distinguishes 8 kinds of information

We will study about the amount of information in the 7th lecture

Conversion

from binary to decimal number

- Calculate the sum of the corresponding power two of weight value for all digits 1 in binary number. (the weight of the left-most digit is 0, the next one is 1...)



$$11001111_{(2)} = 128 + 64 + 0 + 0 + 8 + 4 + 2 + 1 = 207_{(10)}$$

Conversion from decimal to binary (Integers)

- Arrange in reverse order of modulo 2

| | |
|-------------------|---|
| Modulo 2 | |
| 2) <u>207</u> | |
| 2) <u>103</u> ... | 1 |
| 2) <u>51</u> ... | 1 |
| 2) <u>25</u> ... | 1 |
| 2) <u>12</u> ... | 1 |
| 2) <u>6</u> ... | 0 |
| 2) <u>3</u> ... | 0 |
| 2) <u>1</u> ... | 1 |
| 0 ... | 1 |

↑

Arrange in
reverse order

$207_{(10)} = 11001111_{(2)}$

Conversion from decimal to binary (Fraction)

- Arrange the integer part when multiplying fraction with 2

Integer part when multiplying with 2

| | | |
|-------|--|------|
| 0.625 | | |
| × 2 | | |
| 1.25 | | ...1 |
| | | |
| 0.25 | | |
| × 2 | | |
| 0.5 | | ...0 |
| | | |
| 0.5 | | |
| × 2 | | |
| 1.0 | | ...1 |

arrange

$$0.625_{(10)} = 0.101_{(2)}$$

Hexadecimal number

- It's hard to read the binary number once its number of digits increases.
- One digit of hexadecimal number is combined by 4 digits of binary number.

| | | | | | | | | | | | | | | | | |
|---------------------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| Decimal number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Hexadecimal number | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |

Used for character coding (lecture 5), displaying colors in web page (lecture 9), programming, etc.

Conversion

from binary to hexadecimal number

- From the left-most digit, divide to groups that contains each four digits.
- Convert each 4-digit binary number group to decimal number, and then convert it to hexadecimal.

| | | |
|-----|------|------|
| 100 | 1110 | 0110 |
| 4 | 14 | 6 |
| 4 | E | 6 |

$$10011100110_{(2)} = 4E6_{(16)}$$

Conversion

from hexadecimal to decimal number

- Calculate the sum of (weight of each digit * digit value)
- The left-most digit is 16^0 , the next one is 16^1 ...

| | | |
|--|---|--|
| <div style="border: 1px solid black; width: 60px; height: 60px; display: flex; flex-direction: column; align-items: center; justify-content: center;"> 4 16² </div> | <div style="border: 1px solid black; width: 60px; height: 60px; display: flex; flex-direction: column; align-items: center; justify-content: center;"> E(=14) 16¹ </div> | <div style="border: 1px solid black; width: 60px; height: 60px; display: flex; flex-direction: column; align-items: center; justify-content: center;"> 6 16⁰ </div> |
| ⋮ | ⋮ | ⋮ |
| $4E6_{(16)} = 4 \times 256 + 14 \times 16 + 6 \times 1 = 1254_{(10)}$ | | |

$$4E6_{(16)} = 1254_{(10)}$$